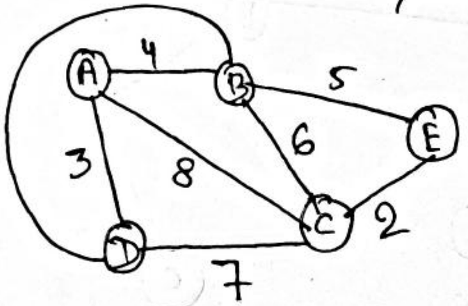
Minimum Spanning Tree

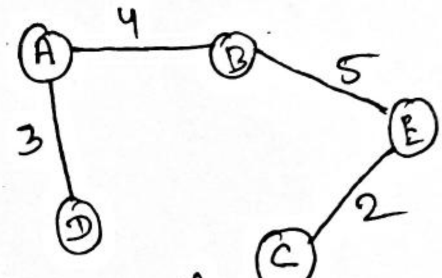
Let $G = \langle V, E \rangle$ is a connected weighted undirected graph.

Spanning tree of a graph consists of all vertices and some of the edges, so that the graph does not contain a cycle.

ex:

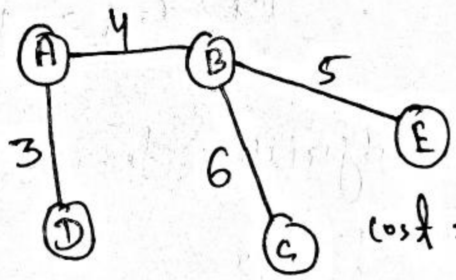


Spanning Tree:



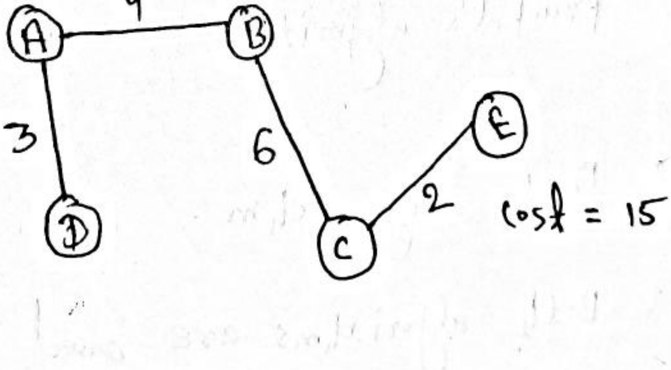
cost = 14

Spanning Tree:



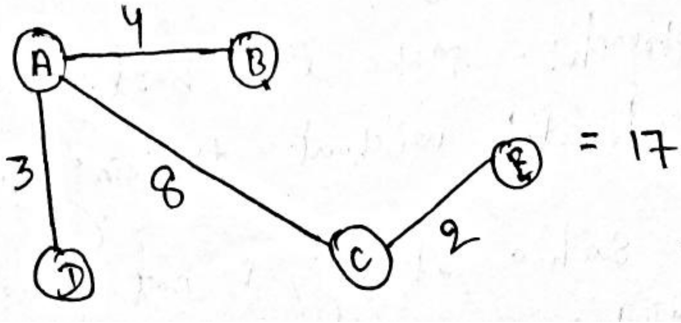
cost = 18

Spanning Tree:



cost = 15

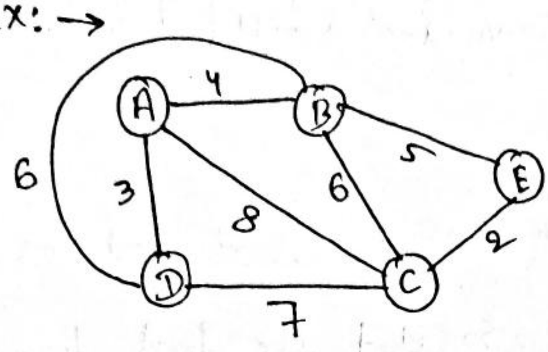
Spanning Tree:



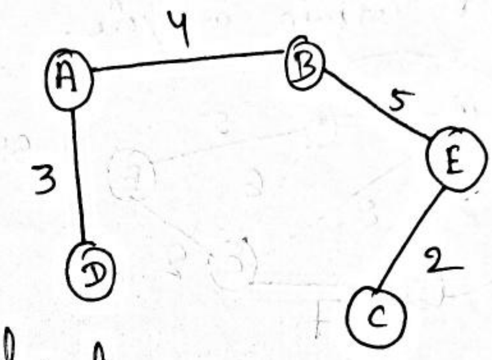
= 17

Minimum Spanning Tree: Spanning Tree of minimum cost/weight/length.

ex: →



Minimum Spanning Tree:



Total cost = 14

There are two algorithms for solving the minimum spanning-tree Problem.

↳ Kruskal's algorithm

↳ Prim's algorithm.

Note: Both algorithms are based on greedy approach.

Greedy approach: - Make the best ~~choice~~ avilable choice at each step without thinking of the future

Note → Such a strategy is not generally gauranteed to find globally optimal solutions to problems.

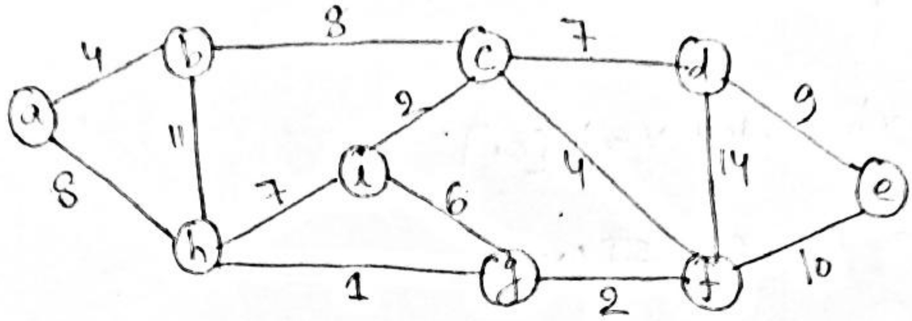
Kruskal's algorithm

MST - Kruskal(G, w)

1. $A \leftarrow \phi$
2. for each vertex $v \in V[G]$
3. do MAKE-SET(v)
4. Sort the edges of E into nondecreasing order by weight w.
5. for each edge $(u,v) \in E$, taken in nondecreasing order by weight.
6. do if FIND-SET(u) $\neq$ FIND-SET(v)
7. then $A \leftarrow A \cup \{(u,v)\}$
8. UNION (u,v)
9. return A

Kruskal's algorithm: Kruskal's algorithm are a greedy algorithm that finds a MST for a connected weighted undirected graph.

Example:→



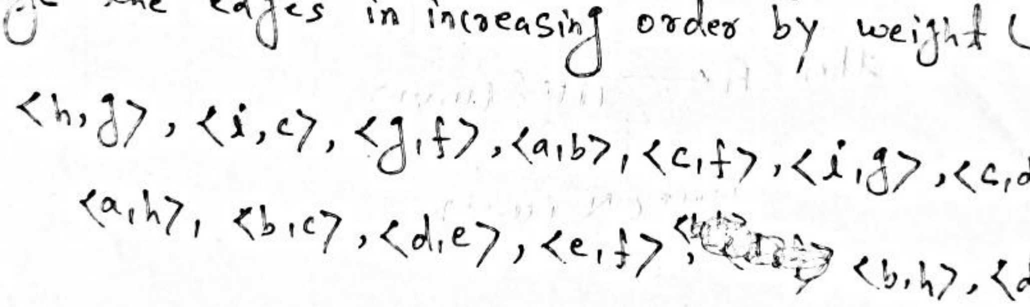
Initially $A \leftarrow \phi$

$\{a\}, \{b\}, \{c\}, \{d\}, \{e\}, \{f\}, \{g\}, \{h\}, \{i\}$ these are the sets of vertices each containing one vertex.
Decompose the edges in increasing order by weight (cost)

$\langle h,g\rangle, \langle i,c\rangle, \langle g,f\rangle, \langle a,b\rangle, \langle c,f\rangle, \langle i,g\rangle, \langle c,d\rangle, \langle h,i\rangle,$
$\langle a,h\rangle, \langle b,c\rangle, \langle d,e\rangle, \langle e,f\rangle, \langle b,h\rangle, \langle d,f\rangle.$

STEP 1:

(a)

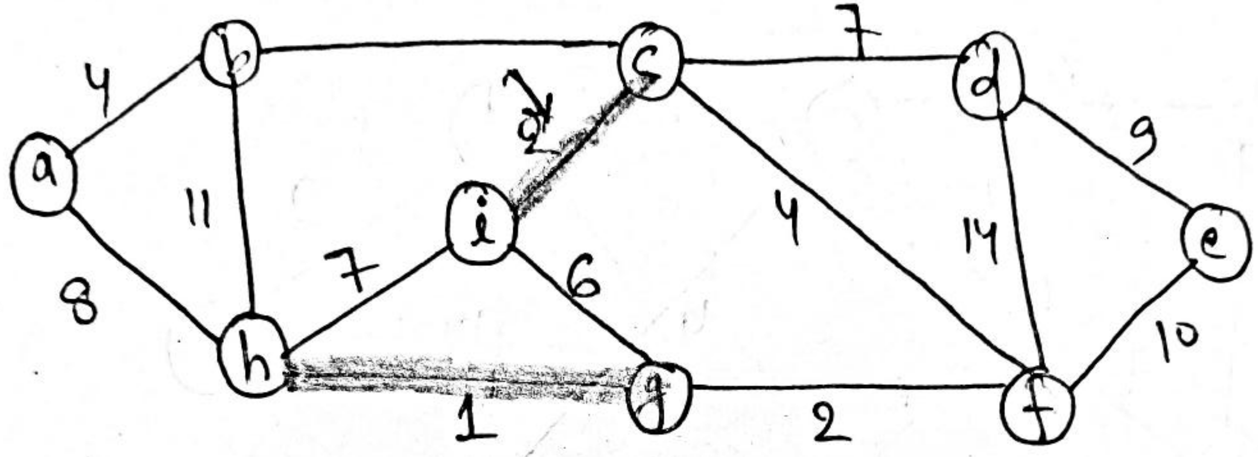


first take edge $\langle h,g\rangle$

and sets are: $\{a\}, \{b\}, \{c\}, \{d\}, \{e\}, \{f\}, \{g,h\}, \{i\}$

$A = \{\langle h,g\rangle\}$

b)



Take edge $\langle i, c \rangle$ in which vertex $i \in$ set$\{i\}$

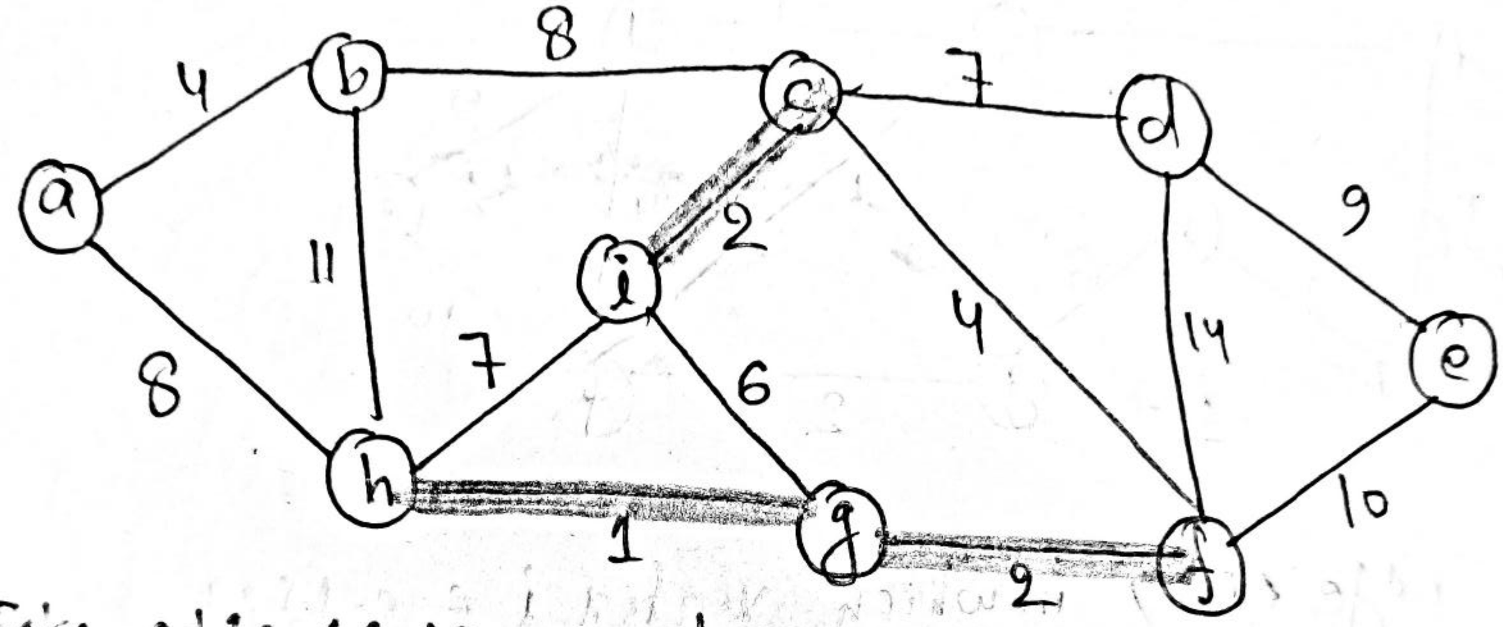
and vertex $c \in$ set$\{c\}$

So, FIND SET$(i) \neq$ FIND-SET(c)

Now $A = \{\langle h, g \rangle, \langle i, c \rangle\}$

Sets are : $\{a\}, \{b\}, \{i, c\}, \{d\}, \{e\}, \{f\}, \{g, h\}$.

c)



Take edge $\langle g, f \rangle$ according to the non-decreasing order

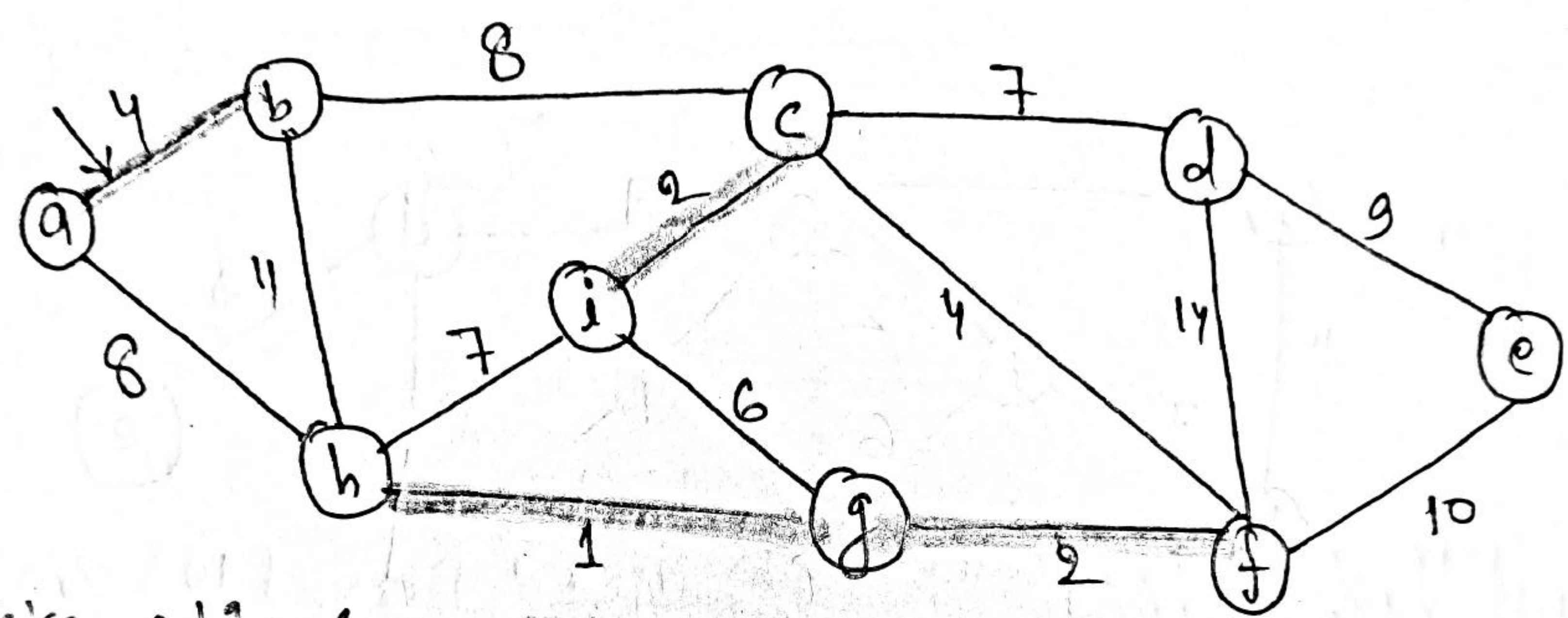
vertex $g \in \{g, h\}$ and $f \in \{f\}$

So, FIND-SET$(g) \neq$ FIND-SET(f).

Now $A = \{\langle h, g\rangle, \langle i, c\rangle, \langle g, f\rangle\}$

Sets are: $\{a\}, \{b\}, \{i, c\}, \{d\}, \{e\}, \{f, g, h\}$

d)



Take edge $\langle a,b \rangle$ according to the non-decreasing order.

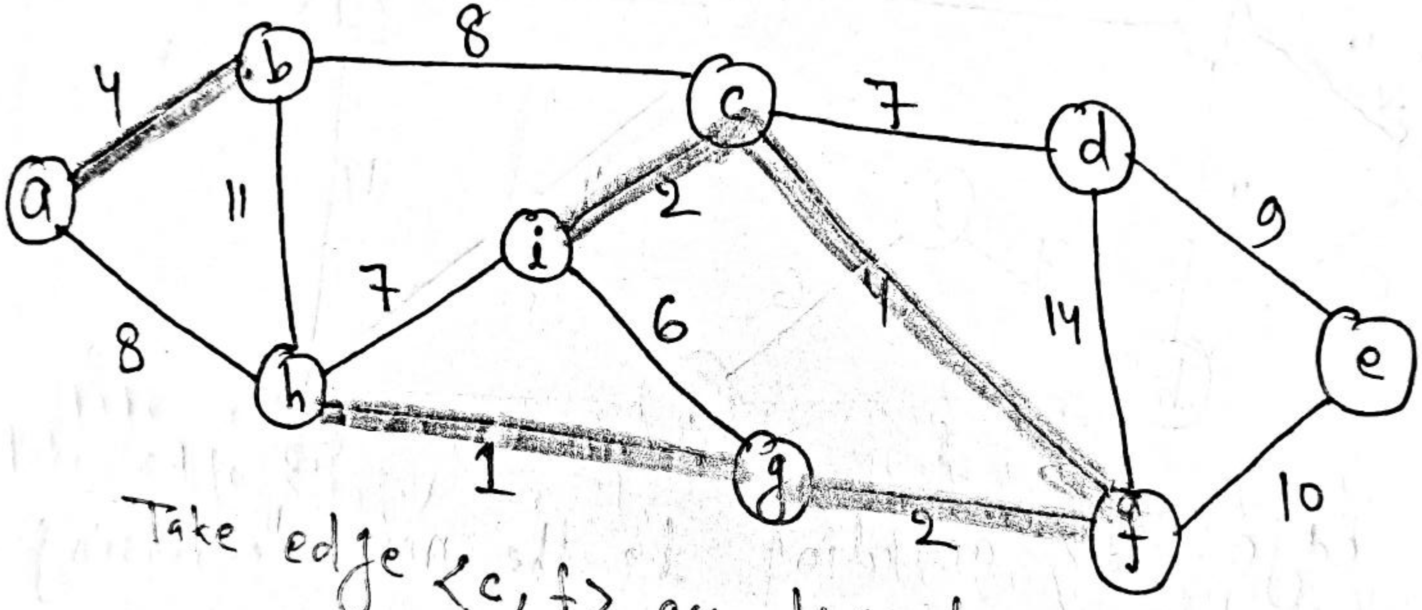
vertex $a \in \{a\}$ and vertex $b \in \{b\}$

So, FIND-SET(a) $\neq$ FIND-SET(b)

Now $A = \{\langle h,g \rangle, \{i,c\rangle, \langle g,f \rangle, \langle a,b \rangle\}$

Sets are: $\{a,b\}, \{i,c\rangle, \{d\}, \{e\}, \{f,g,h\}$

e)



Take edge $\langle c, f \rangle$ according to the non-decreasing order

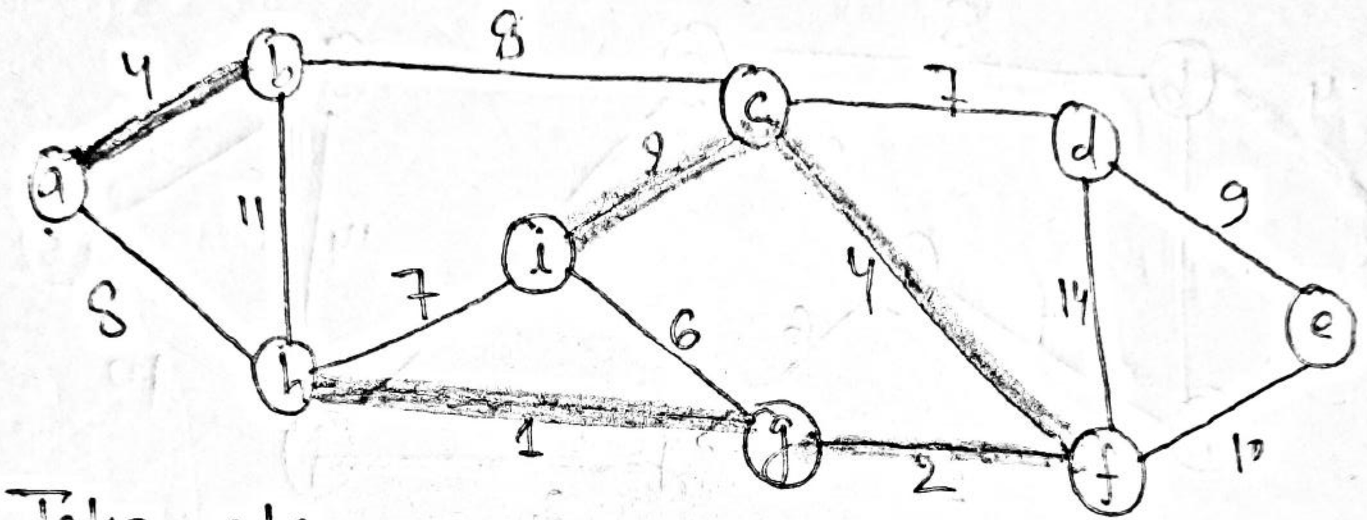
vertex $c \in \{i, c\}$ and $f \in \{f, g, h\}$

So, FIND-SET$(c) \neq$ FIND-SET(f)

$A = \{\langle h, g \rangle, \langle i, c \rangle, \langle g, f \rangle, \langle a, b \rangle, \langle c, f \rangle\}$

Sets are : $\{a, b\}$, $\{i, c, f, g, h\}$, $\{d\}$, $\{e\}$.

4).



Take edge $\langle i, g \rangle$ according to the non-decreasing order

vertex $i \in \{i, c, f, g, h\}$ and $g \in \{i, c, f, g, h\}$

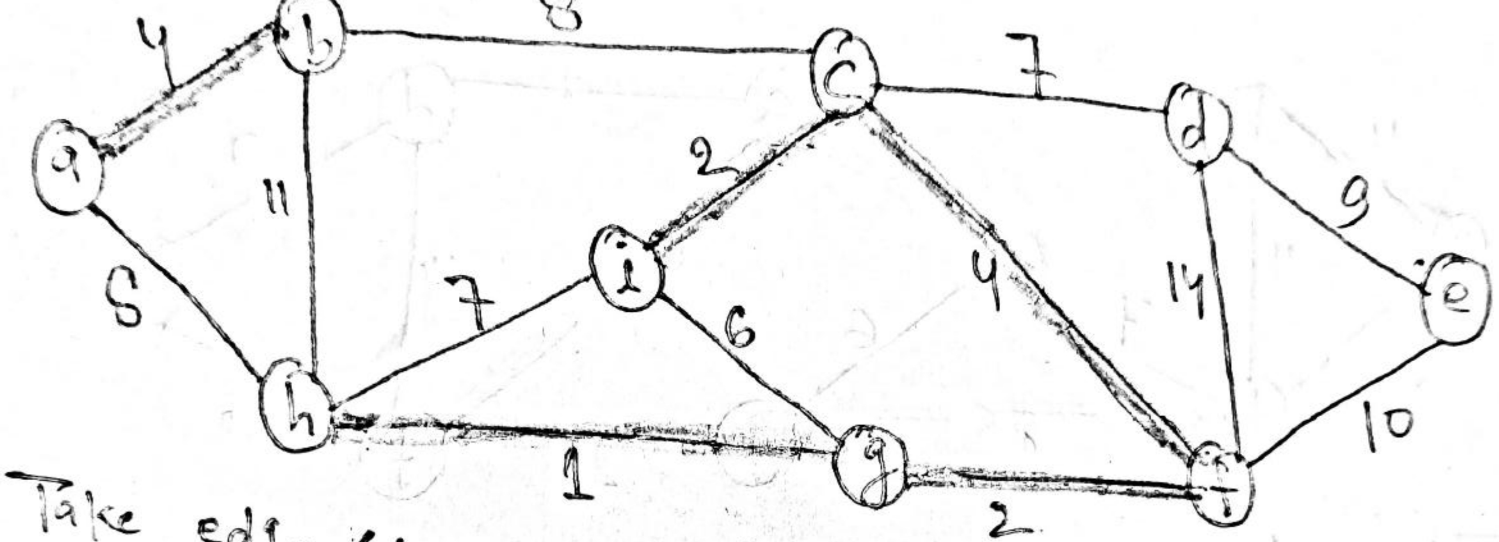
So, FIND-SET(i) = FIND-SET(g)

edge $\langle i, g \rangle$ will not be added to A

$A = \{\langle h, g \rangle, \langle i, c \rangle, \langle g, f \rangle, \langle a, b \rangle, \langle c, f \rangle\}$

Sets are : $\{a, b\}, \{i, c, f, g, h\}, \{d\}, \{e\}$

f)



Take edge $\langle h,i \rangle$ according to the non-decreasing order

vertex $h \in \{i,c,f,g,h\}$ and $i \in \{i,c,f,g,h\}$

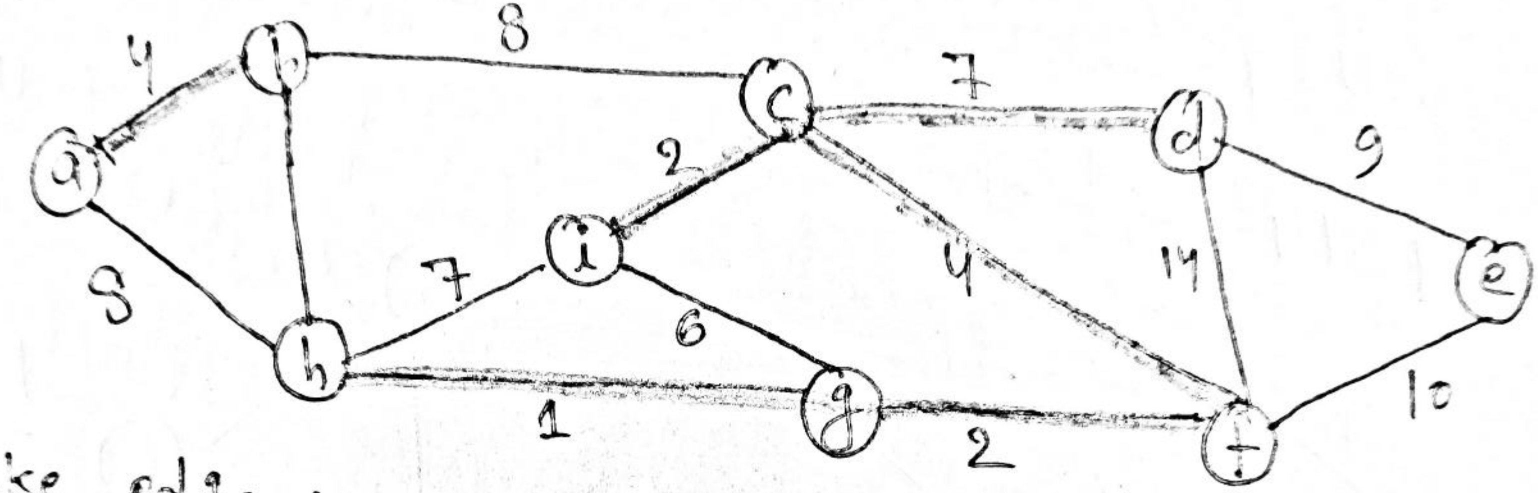
FIND-SET(h) = FIND-SET(i)

So, edge $\langle h,i \rangle$ will not be added to set A

$A = \{\langle h,g \rangle, \langle i,c \rangle, \langle g,f \rangle, \langle a,b \rangle, \langle c,f \rangle\}$

Sets are : $\{a,b\}, \{i,c,f,g,h\}, \{d\}, \{e\}$.

h)



Take edge $\langle c,d \rangle$ according to the non-decreasing order

vertex $c \in \{i, c, f, g, h\}$ and $d \in \{d\}$

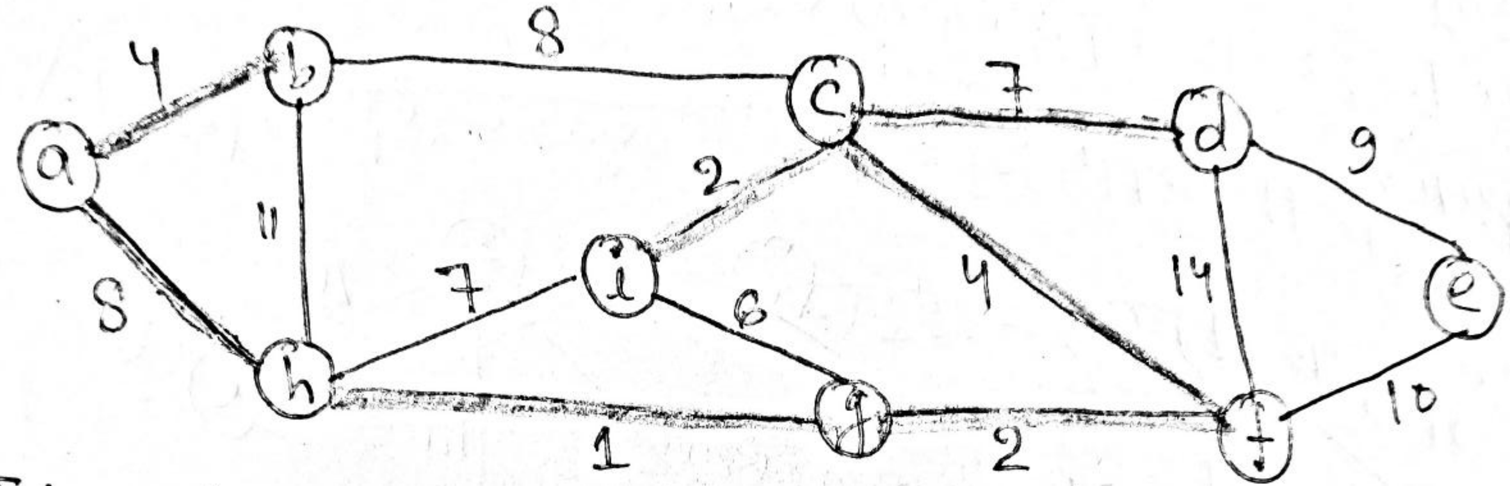
$\text{FIND-SET}(c) \neq \text{FIND-SET}(d)$

So, edge $\langle c,d \rangle$ will be added to the set A

$A = \{\langle h,g \rangle, \langle i,c \rangle, \langle g,f \rangle, \langle a,b \rangle, \langle c,f \rangle, \langle c,d \rangle\}$

Sets are : $\{a,b\}, \{i, c, f, g, h, d\}, \{e\}$

(i)



Take edge $\langle a,h \rangle$ according to the non-decreasing order
vertex $a \in \{a,b\}$ and $h \in \{i,c,f,g,h,d\}$

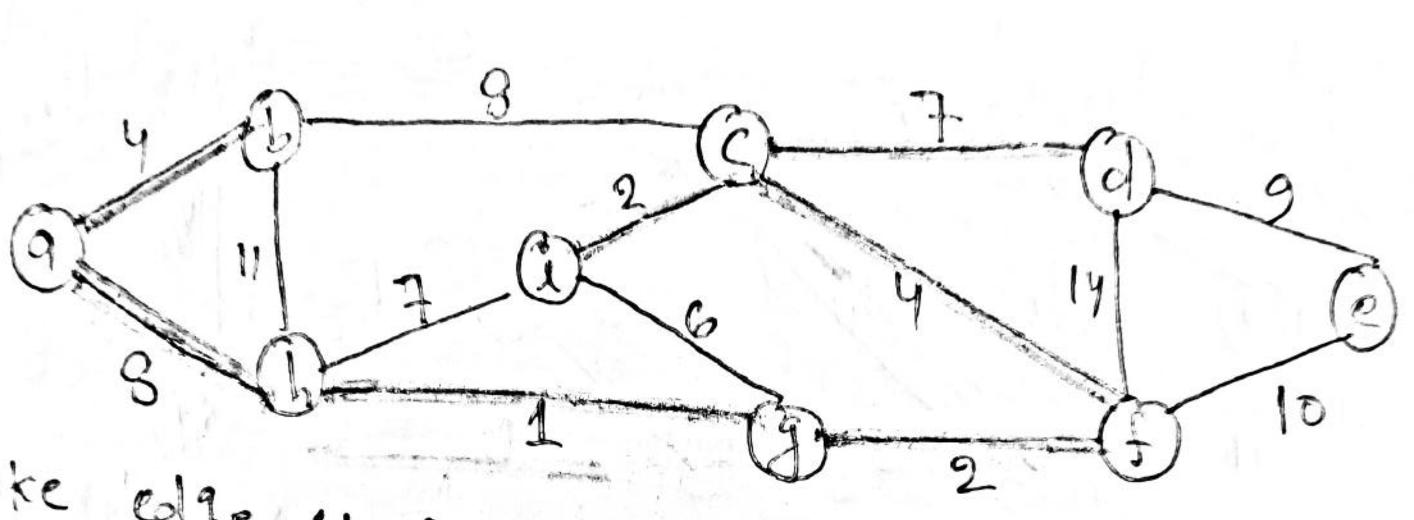
FIND-SET(a) $\neq$ FIND-SET(h)

So, edge $\langle a,h \rangle$ will be added to the set A

$A = \{\langle h,g \rangle, \langle i,c \rangle, \langle g,f \rangle, \langle a,b \rangle, \langle c,f \rangle, \langle c,d \rangle\}$

Set are: $\{a,b,i,c,f,g,h,d\}, \{e\}$.

$\langle j \rangle$



Take edge $\langle b,c \rangle$ according to the non-decreasing order

vertex $b \in \{a,b,i,c,f,g,h,d\}$ and $c \in \{a,b,i,c,f,g,h,d\}$

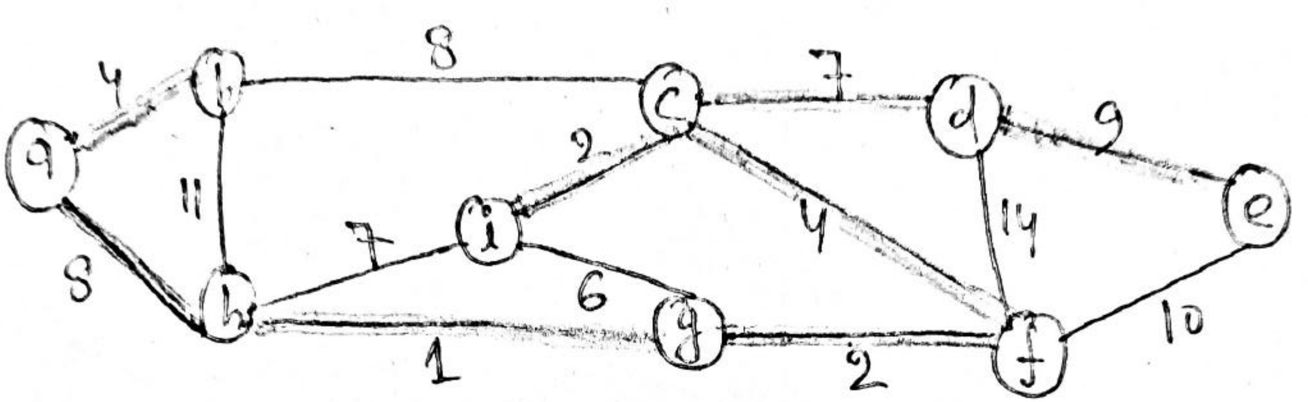
FIND-SET (b) = FIND-SET (c)

So, edge $\langle b,c \rangle$ will not be added to the set A

$A = \{\langle h,g \rangle, \langle i,c \rangle, \langle g,f \rangle, \langle a,b \rangle, \langle c,f \rangle, \langle c,d \rangle\}$

Sets are: $\{a,b,i,c,f,g,h,d\}, \{e\}$

<k>



Take edge <d,e> according to the non-decreasing order

vertex $d \in \{a, b, i, c, g, h, d\}$ and $e \in \{e\}$

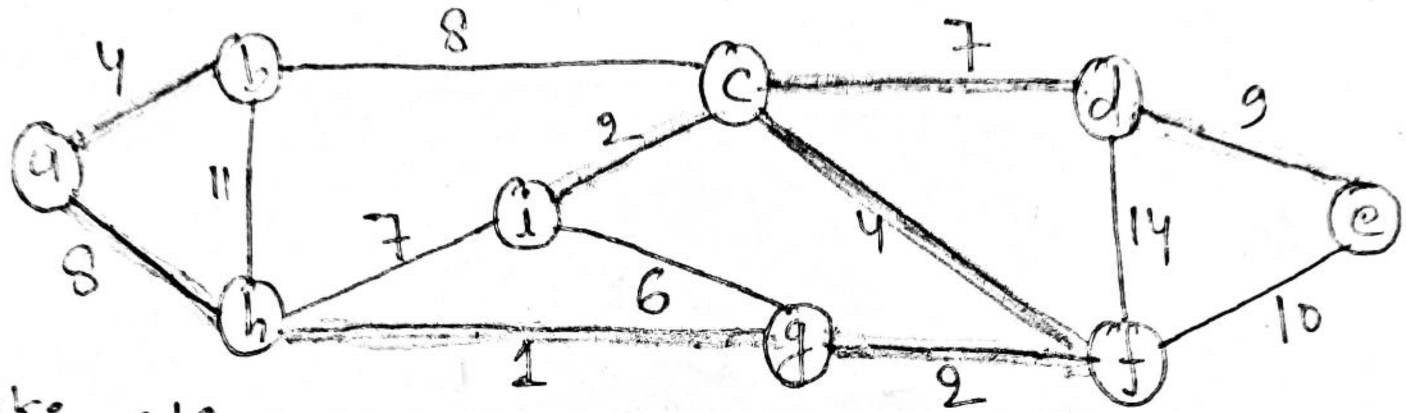
FIND-SET (d) $\neq$ FIND-SET (e)

So, edge <d,e> will be added to the set A

$A = \{\langle h,g\rangle, \langle i,c\rangle, \langle g,f\rangle, \langle a,b\rangle, \langle c,f\rangle, \langle c,d\rangle, \langle d,e\rangle\}$

Sets are: $\{a, b, i, c, g, h, d, e\}$

⟨L⟩



Take edge ⟨e,f⟩ according to the non-decreasing order
vertex $e \in \{a,b,c,d,e,f,g,h,i\}$ and $f \in \{a,b,c,d,e,f,g,h,i\}$

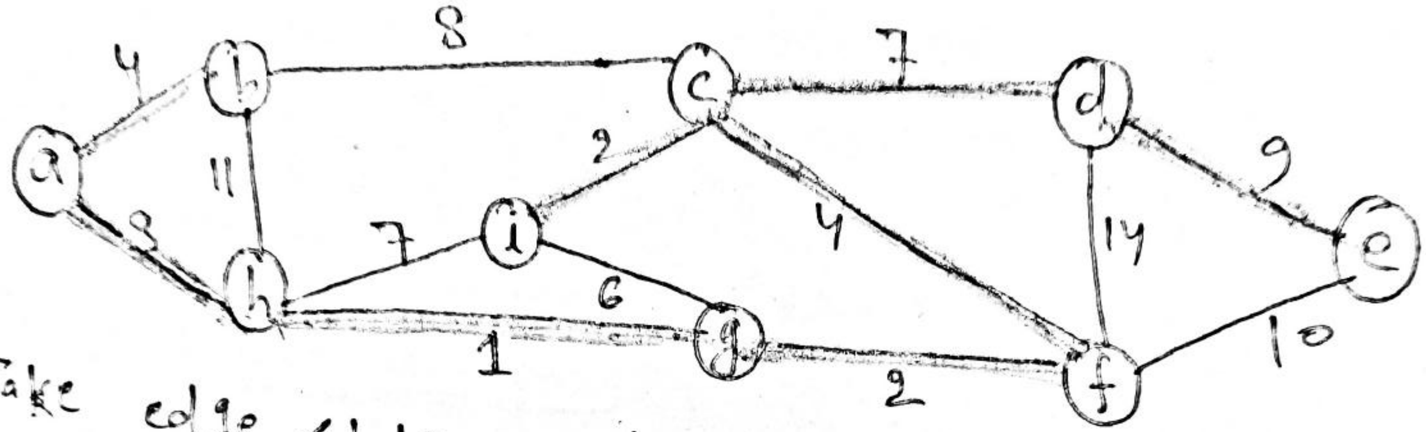
FIND-SET(e) = FIND-SET(f)

So, edge ⟨e,f⟩ will not be added to the set A

$A = \{\langle h,g\rangle, \langle i,c\rangle, \langle g,f\rangle, \langle a,b\rangle, \langle c,f\rangle, \langle c,d\rangle, \langle d,e\rangle\}$

Sets are : $\{a,b,c,d,e,f,g,h,i\}$

<M>



Take edge <b,h> according to the non-decreasing order vertex $b \in \{a,b,c,d,e,f,g,h,i\}$ and $h \in \{a,b,c,d,e,f,g,h,i\}$

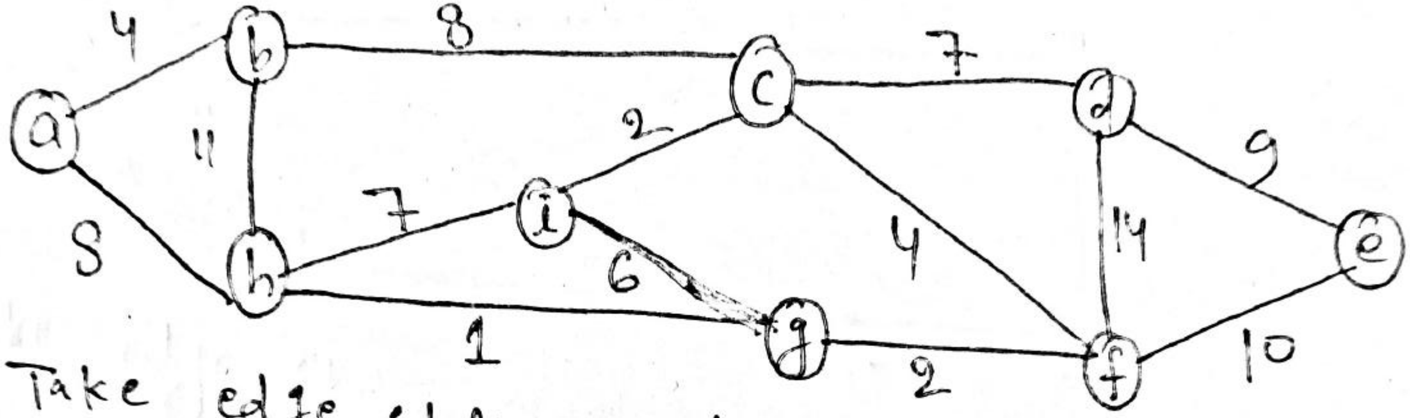
FIND-SET(b) = FIND-SET(h)

So, edge <b,h> will not be added to the set A

$A = \{ \langle h,g\rangle, \langle i,c\rangle, \langle g,f\rangle, \langle a,b\rangle, \langle c,f\rangle, \langle c,d\rangle, \langle d,e\rangle \}$

Sets are : $\{a,b,c,d,e,f,g,h,i\}$

⟨N⟩



Take edge $\langle d,f \rangle$ according to the non-decreasing order

vertex $d \in \{a,b,c,d,e,f,g,h,i\}$ and $f \in \{a,b,c,d,e,f,g,h,i\}$

FIND-SET(d) = FIND-SET(f)

So, edge $\langle d,f \rangle$ will not be added to the set A

$A = \{\langle h,g \rangle, \langle i,c \rangle, \langle g,f \rangle, \langle a,b \rangle, \langle c,f \rangle, \langle c,d \rangle, \langle d,e \rangle\}$

Sets are: $\{a,b,c,d,e,f,g,h,i\}$

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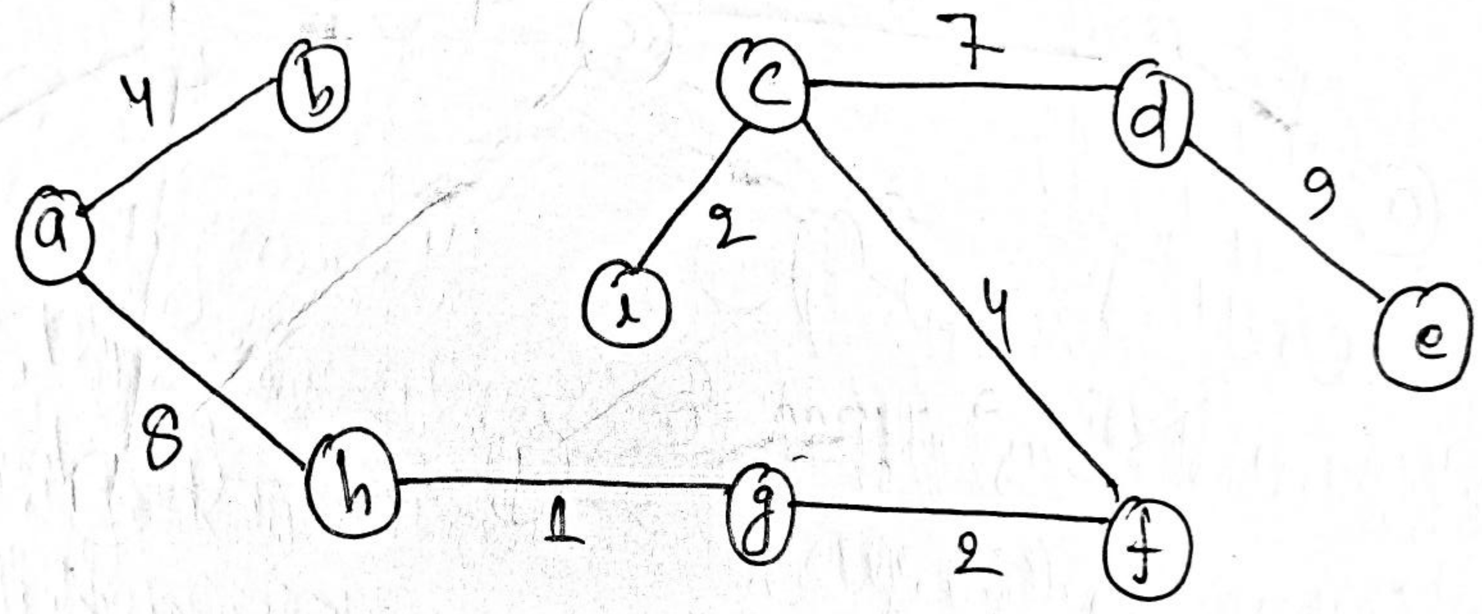

Minimum Cost = 37.

(Minimum Spanning Tree).